Microstrip Characteristic Impedance

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Abstract—**A recent paper by Bianco *et al.* [1] gives five definitions of microstrip characteristic impedance which do not have the same frequency dependence. This paper shows that the most widely used definition can be based on wave-impedance and is a unique, unambiguous characteristic impedance for microstrip, consistent with characteristic impedance definitions of other waveguiding structures.**

BIANCO *et al.* [1] have recently defined microstrip characteristic impedance in five ways, based on three definitions of voltage and one of current. They find that the different impedances do not have the same functional dependence on frequency. Since they do not discuss the merits of the differing definitions, the reader may assume that all are equally valid representations of microstrip characteristic impedance.

This paper is intended to point out that the well-known concept of wave-impedance can be applied to microstrip as well as to TEM transmission lines and waveguides and that it yields a microstrip characteristic impedance which is unique and unambiguous in its dependence on frequency. This is also the microstrip impedance definition most widely accepted today.

Forty years ago, Schelkunoff [2] published a discussion of the impedance concept and extended it to include wave-impedance. Subsequently, Schelkunoff [3] applied the idea of wave-impedance to transmission lines. Wave-impedance Z_w is defined at each point in the cross section of a transmission line by

$$Z_w = \frac{E_y}{H_x} = -\frac{E_x}{H_y} \tag{1}$$

where E and H are the electric and magnetic field components transverse to the direction of propagation. For a specific mode or wave type in isotropic homogeneous media, impedance definitions based on voltage, current, and power are directly proportional to wave-impedance, which is the same at every point on a surface transverse to the direction of propagation. This is well understood in the case of TEM transmission lines and hollow waveguides.

It is necessary to show that wave-impedance is also the same at every point of the cross section of microstrip, which is inhomogeneous. Appropriate transverse field expressions based on the LSE model [4] of microstrip are given by Bianco *et al.* [1]

$$H_x = -\gamma^2 A_i \cosh(\gamma_i \chi_i) \tag{2}$$

$$E_y = j\omega\mu_0 \gamma A_i \cosh(\gamma_i \chi_i) \tag{3}$$

where γ is the longitudinal propagation constant of the wave and γ_i is the transverse propagation constant, for which the subscript i indicates the necessity of using the value appropriate to the region, air or dielectric, of the model. Combining (1), (2), and (3) gives

$$Z_w = -j\frac{\omega\mu_0}{\gamma} \tag{4}$$

which is independent of the region. Further,

$$\gamma = -j\frac{\omega\sqrt{\epsilon_e}}{c} \tag{5}$$

where ϵ_e is the effective dielectric constant [4], a function of frequency, and c is the speed of light in vacuum. Thus

$$Z_w = \frac{\eta_0}{\sqrt{\epsilon_e}} \tag{6}$$

in which η_0 is the impedance of free space, 376.7 Ω.

Equation (6) shows that Z_w is independent of the region, air or dielectric. The proportionality factor between wave-impedance and total impedance is established by evaluation at zero frequency, where all definitions agree, using MSTRIP [5] or a similar static solution. Thus the microstrip characteristic impedance Z_c is given by

$$Z_c = Z_0\sqrt{\frac{\epsilon_{e0}}{\epsilon_e}} \tag{7}$$

in the notation of [4].

It can be concluded then that only one of the definitions of microstrip characteristic impedance given by Bianco *et al.* (Z_1 of eq. (10) in [1]) is consistent with the more general approach based on wave-impedance, which gives a unique result.

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